Performance Analysis of RACH procedure with Beta Traffic-Activated Machine-Type-Communication

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***Abstract*— Machine-Type-Communication (MTC) is a key enabler for a variety of novel smart systems, such as smart grid, eHealth, Intelligent Transport System (ITS), and smart city, opening the area of the cyber physical systems. These systems may require the use of a huge number of MTC devices, which will put a great pressure on the whole network, i.e. Radio Access Network (RAN) and Core Network (CN) parts, resulting in the shape of congestion and system overload. Aiming at better evaluating the network performance under the existence of MTC traffic and also the effectiveness of the congestion control methods, the 3rd Generation Partnership Project (3GPP) group has proposed two traffic models: Uniform Distribution (over 60 s) and Beta Distribution (over 10 s). In this paper, a recursive operation-based analytical model, namely General Recursive Estimation (GRE), for modeling the performance of RACH procedure in the existence of MTC with Beta traffic is proposed. In order to show the effectiveness of our analytical model GRE, many metrics have been considered, such as the total number of MTC devices in each Random Access (RA) slot, the number of success MTC devices in each RA slot, and the Cumulative Distribution Function (CDF) of preamble transmission. Numerical results demonstrate the accuracy of GRE. Moreover, our model GRE could be used to model the performance of RACH procedure with any type of traffic.**

I. INTRODUCTION

In the near future, our environment will be surrounded by a huge number of connected objects, building the so-called Internet of Thing (IoT). Machine-Type-Communication(MTC), or alternatively Machine-to-Machine (M2M), can be considered as a cornerstone of the vision of IoT. MTC can be viewed as autonomous devices connecting to the network without, or with a little, human intervention. Some studies foresee that there would be roughly 50 billion of MTC devices by 2020 [1]. However, the number of MTC devices might be even more than what is expected, especially after the introduction of the new type of communication, namely Visible Light Communication (VLC) [2], [3], [4], [5], that may open the door for more and more applications and thus much higher number of MTC devices. Nowadays, a vast number of MTC applications, comprising a large number of fields, have already been deployed. Some of these applications are Healthcare, Intelligent Transport System (ITS), smart metering and smart grids, public safety (PS), forming the so-called smart city.

However, introducing this diversity of applications, that rely on a huge number of MTC devices, would put very high pressure on the current networks, especially on the cellular mobile networks as it is considered of best candidate for enabling MTC connections. Deploying the expected number of MTC devices in the cellular mobile networks would face many challenges. As an example, the current cellular mobile networks are not designed in a way to support this huge number of MTC devices. Although it is expected that the traffic per MTC device would be considerably low, roughly $1Kbyte$, the aggregated traffic from all the MTC devices would be very high. However, it is expected that MTC traffic would increase 24 times by 2017 compared to 2012, and the total traffic volume, in the wireless communication systems, would be increased 1000 times compared to today's traffic volume [6], [7]. Deploying applications that need to employ a high number of MTC devices associated with the huge amount of data/control traffic will certainly cause congestion and system overload in the whole network, i.e. in the Radio Access Network (RAN) part and the Core Network (CN) part. This effect of MTC on the network may engender intolerable delay, packet loss, or even service unavailability for all the terminals in the network. Intolerable delay means that the terminal, more precisely MTC devices, would take a long time to get access the network, and thus a long period in the active state. The longer period in the active state, the higher consumption of the power and the shorter the battery life. It should be noted that the power consumption is a very important issue in the context of MTC as the devices would be equipped with a battery that would not be changed for a long period, e.g. 4 years, or even more.

Separating the Random Access CHannel (RACH) resources, dynamic allocation of RACH resources, Access Class Barring (ACB) methods are some good examples of the methods used to avoid (alleviate) the congestion's problem. The aforementioned methods are considered as Push based methods as the RACH procedure is initiated by the terminals rather than the network [8]. Separation of RACH resources consists in dividing the available resources into two groups: one for MTC traffic and one for Non-MTC traffic. Dynamic allocation of RACH resources can be viewed as an improvement of the RACH separation method. However, it can be applied only when the network is aware about the time when MTC devices have traffic to send. Regarding the ACB method, it classifies the terminals into many classes [9]. But ACB method could be used only when one or more classes are well defined, where each class has its own backoff parameters for the RACH procedure. Accordingly, ACB method ensures the priority between MTC classes and classical traffic.

Another type of congestion control methods is Pull based approach, where the network initiates the connection. Under this approach, many methods can be found, such as Paging and Group Paging (GP) methods. In the paging method, the network, e.g. eNB, will send to the intended MTC device a paging message identified by its ID. This method is a rational one when applied to a relatively low number of devices, but it becomes infeasible when there is a large number of devices, as this is the case for MTC applications. Th solution of this problem is the GP method, where one paging message is sent to all the intended MTC devices addressed by Group ID (GID), where all the concerned devices are grouped to this group. One of the GP method's improvements in the literature is the one proposed in [10], where a terminal ID-based scheduling, namely Controlled Distribution of Resources (CDR), is used. The CDR method highly improves the performance of GP method. However, it targets only the MTC devices in the $RRC\_CONNECTED$ mode. Another improvement is the method presented in [11], namely Traffic Spreading For Group Paging (TSFGP). The advantage of this method is that it highly improves the performance in the case of Group Paging, regardless the mode of the MTC devices, i.e. whether they are RRC_IDLE or $RRC\_CONNECTED$.

However, in order to evaluate the network performance under different access intensities and also show the effectiveness of the congestion control methods for MTC applications, we should first define a good traffic model that characterize the behavior of this type of devices. 3GPP has identified two traffic models: Uniform Distribution (over 60 s) as a realistic scenario where the MTC devices access the network uniformly over a certain period (non-synchronized traffic), and Beta Distribution (over 10 s) as an extreme scenario where the MTC devices are activated in a highly synchronized manner during certain period (synchronized traffic) [8]. In this paper, we propose an analytical model, namely General Recursive Estimation (GRE), for modeling the performance of RACH procedure in the existence of MTC traffic with Beta Distribution. To our knowledge, this is the first time that RACH procedure with Beta traffic, in the context of MTC, is modeled. GRE is based on the recursive estimation of the number of MTC devices, transmitting their preamble for the i^{th} time, in each RA slot.

The rest of the paper is organized as follows. Section II gives an overview about the Random Access CHannel (RACH) procedure as specified in Long Term Evolution (LTE) and LTE-Advanced (LTE-A) networks. In section III, system model used in our study and our proposed analytical model, namely General Recursive Estimation (GRE), are elaborated. The evaluation of GRE's performance is presented in section IV. Finally, conclusions are introduced in section V.

II. BACKGROUND: RACH PROCEDURE

Generally speaking, a terminal in Long Term Evolution (LTE) and LTE-Advanced (LTE-A) networks can be in one of two modes: Radio Resource Control (RRC) Idle mode or RRC Connected mode. In the RRC Idle mode, the terminal cannot neither receive nor transmit specific data, while it can transmit/receive specific data in the RRC Connected mode. When a terminal in the Idle mode needs to access the network, the first thing to do is the RACH procedure. There are two forms of RACH procedure: Contention-based and Contention-free RACH procedures. The Contention-based procedure is used, in general, when the terminal tries to connect the network, e.g. to establish the connection or to restore the Uplink synchronization. On the other hand, the Contention-free procedure is used when the connection is initiated by the network, e.g. when there is handover or Downlink data arrival. The RACH procedure consists of the following steps (as illustrated in figure 1):

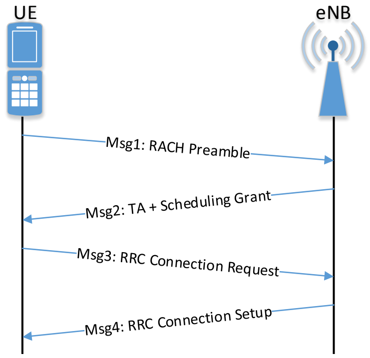

Fig. 1: Random Access CHannel (RACH) procedure

1) *Random Access Preamble Transmission (Msg1):* This step consists of the transmission of a preamble, where the terminal, User Equipment (UE) or MTC, randomly chooses one out of the available preambles. Because of the randomness, we may encounter the case that more than one terminal choose the same preamble, and thus causing a collision. In this case, all the terminals having chosen the same preamble will back off and retransmit the preamble later. Another objective of this step is to adjust the Uplink synchronization, where the eNB will estimate the transmission timing of the terminal that is used to adjust the synchronization.
2) *Random Access Response Reception (Msg2):* After transmitting the preambles, the terminal monitors the Physical Downlink Control CHannel (PDCCH) during certain interval in order to receive the response message. This interval is Random Access Response (RAR) window. It should be noted that the maximum number of responses (N_{ACK}) during the RAR window is:

$$N_{ACK} = N_{RAR} W_{RAR}$$

where N_{RAR} is the maximum number of RARs per a response message. The response message contains many parameters, such as the Timing Advanced (TA) used to adjust the uplink synchronization and the terminal's identifier Temporary Cell-Radio Network Temporary Identifier (TC-RNTI). TC-RNTI is the temporary ID of the terminal within the cell, and it can be later promoted to C-RNTI if the terminal has not yet a one. This message also contains on the UpLink (UL) resources to be used by the terminal in the next step. By the reception of the RAR message, this is the end of the Contention-free procedure, while the terminals with a Contention-based one will continue to the next step.
3) *RRC Connection Request (Msg3):* After the reception and processing the message Msg2, the terminal will send the message Msg3 to request RRC connection from the

network. This message also contains on the ID of the terminal.

4) *RRC Connection Setup (Msg4):* This message, send by the network, is a response message to the precedent one. Another objective of this message is to solve the problem when more than one terminal choose the same preamble and the network successfully receives this preamble, thus having the same temporary ID, i.e. TC-RNTI. To solve this problem, each terminal receives the message Msg4 will compare the ID in this message with that one transmitted in Msg3. Only the terminal that observes a match between the two will consider that the RACH procedure has been successfully finished, while the others will back off and then retry transmitting the preamble after the expiration of the backoff timer.

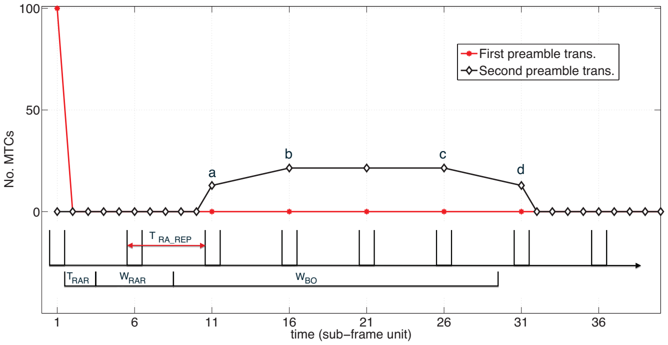

Fig. 2: Number of MTC devices at each RA slot for the first and second preamble transmission for $R = 54$, and $M/N = 100$ [11]

III. GENERAL RECURSIVE ESTIMATION (GRE)

A. System Model

In this study, we assume that the MTC traffic is generated according to Beta distribution as specified by 3GPP [8]. It is assumed that all the MTC devices fall within the coverage of just one Base Station, i.e. eNB. We assume that there will be just one type of traffic, i.e. Beta traffic, during the considered interval, which is equal to 10 s. Regarding the resources, the eNB reserves R random access preambles. However, the random access resources are determined in terms of Random Access Opportunity (RAO). RAOs can be defined as the number of available preambles multiplied by the number of frequency bands dedicated for the random access. For the sake of simplicity, we consider that there is just one frequency band, and thus the number of RAOs is equal to the number of available preambles. Generally, for each preamble transmission the MTC device could take up to $(T_{RAR} + W_{RAR} + W_{BO})$ sub-frames before retrying the transmission of preamble, as illustrated in figure (2). Therefore, the number of RA slots required in our study, i.e. Beta Distribution, will be equal to:

$$I_{ra} = \left\lceil \frac{I_\beta}{T_{RAREP}} \right\rceil + (N_{PT\_max} - 1) \times \left\lceil \frac{T_{RAR} + W_{RAR} + W_{BO}}{T_{RAREP}} \right\rceil \tag{1}$$

where I_β is the interval of Beta Distribution, in a sub-frame unit, that is equal to ($10\ s = 10 * 1000$ sub-frames), and N_{PT_max} is the maximum number of preamble transmission.

B. Analytical Model

Generally, When there are M_i MTC devices contending on the RACH resources at the time (i), the Idle, Success, and Collision probabilities are equal to:

$$P_I(i) = \binom{M_i}{0}\left(\frac{1}{R}\right)^0\left(1-\frac{1}{R}\right)^{M_i} = \left(1-\frac{1}{R}\right)^{M_i} \approx e^{-\frac{M_i}{R}}$$

$$P_S(i) = \binom{M_i}{1}\left(\frac{1}{R}\right)^1\left(1-\frac{1}{R}\right)^{M_i-1} \approx \frac{M_i}{R}e^{-\frac{M_i}{R}}$$

$$P_C(i) = 1 - P_I(i) - P_S(i)$$

where $\binom{m}{k}$ is k-combinations and it is equal to $\frac{m!}{k!(m-k)!}$. The number of success MTC devices, which is equal to the number of preambles having been chosen by only one MTC device, is equal to:

$$M_S(i) = R \times P_S(i) = M_i e^{-\frac{M_i}{R}} \tag{2}$$

However, the total number of MTC devices M_i comprises the MTC devices transmitting their preambles for the first, second, ..., and the N_{PT_max}-th time, and thus the precedent equation can be written by:

$$M_S(i) = \sum_{n=1}^{N_{PT\_max}} M_i[n] e^{-\frac{M_i}{R}} \tag{3}$$

As the detection probability, by the eNB, is equal to $(p_n = 1 - e^{-n})$ for the n^{th} preamble transmission rather than (1), the precedent equation will thus be written by:

$$M_S(i) = \sum_{n=1}^{N_{PT\_max}} M_i[n] e^{-\frac{M_i}{R}} p_n \tag{4}$$

When the number of success MTC devices exceeds the capacity of the network, i.e. N_{ACK}, the equation (4) should be normalized so that the total number of success MTC devices will be equal to N_{ACK}:

$$M_S(i) = \frac{\sum_{n=1}^{N_{PT\_max}} M_i[n] p_n e^{-\frac{M_i}{R}}}{\sum_{n=1}^{N_{PT\_max}} M_i[n] p_n e^{-\frac{M_i}{R}}} \times N_{ACK} \tag{5}$$

From the equations (4) and (5), we can find that the number of success MTC devices for each preamble transmission is equal to:

$$M_{S,n}(i) = \begin{cases} M_i[n] p_n e^{-\frac{M_i}{R}} & ; \text{if } \eta_i \leq N_{ACK} \\ \frac{M_i[n] p_n e^{-\frac{M_i}{R}}}{\eta_i} N_{ACK} & ; \text{otherwise} \end{cases} \tag{6}$$

where $\eta_i = \sum_{n=1}^{N_{PT\_max}} M_i[n] p_n e^{-\frac{M_i}{R}}$. Based on our analysis in [11], we have the following:

$$\begin{aligned} x_a(i) &= i + \left\lceil \tfrac{T_{RAR} + W_{RAR}}{T_{RA\_REP}} \right\rceil \\ x_{bc}(i) &= i + \left\lceil \tfrac{T_{RAR} + W_{RAR}}{T_{RA\_REP}} \right\rceil + k \\ x_d(i) &= i + \left\lfloor \tfrac{T_{RAR} + W_{RAR} + W_{BO}}{T_{RA\_REP}} \right\rfloor + 1 \end{aligned} \tag{7}$$

where $x_a(i)$, $x_{bc}(i)$, and $x_d(i)$ are the order of the RA slots (a), (bc), and (d), respectively, within the backoff interval W_{BO} related to the preamble transmission at the RA slot (i), as illustrated in figure (2). Alternatively, the proportions of the collided MTC devices whose backoff timers expire and retransmit their preambles at the RA slots (a), (bc), and (d) are equal to:

$$\begin{aligned}
\alpha_a &= \frac{\left\lceil\frac{T_{RAR}+W_{RAR}}{T_{RA\_REP}}\right\rceil T_{RA\_REP}-(T_{RAR}+W_{RAR})}{W_{BO}} \\
\alpha_{bc} &= \frac{T_{RA\_REP}}{W_{BO}} \\
\alpha_d &= \frac{T_{RAR}+W_{RAR}+W_{BO}}{W_{BO}} - \\
&\qquad \frac{T_{RA\_REP}}{W_{BO}}\left\lfloor\frac{T_{RAR}+W_{RAR}+W_{BO}}{T_{RA\_REP}}\right\rfloor
\end{aligned} \tag{8}$$

From the equations (7) and (8) and figure (2), we can conclude the following (note that the effect of retransmission Msg3 and Msg4 will be ignored, as assumed by [12]):

$$M_i[n] = \sum_{j=i-k_2}^{i-k_1} \alpha_j M_{j,C}[n-1]\ ; \text{for } n = 2 : N_{NPT_{max}} \tag{9}$$

where $M_{j,C}[k]$ is the number of collided MTC devices corresponding to the preamble transmission at the RA slot (j) for the k^{th} time, α_j can be α_a, α_{bc}, or α_d, and k_1 and k_2 are given by the following equations (by them the effect of the precedent RA slots on the current RA slot, i.e. the slot (i), is determined):

$$\begin{aligned}
k_1 &= \left\lceil\frac{T_{RAR}+W_{RAR}}{T_{RA\_REP}}\right\rceil \\
k_2 &= 1 + \left\lfloor\frac{T_{RAR}+W_{RAR}+W_{BO}}{T_{RA\_REP}}\right\rfloor
\end{aligned} \tag{10}$$

It is worth noting that k_1 and k_2 are determined directly from x_a and x_d, respectively. However, the equation (9) can be written by the following form:

$$\begin{aligned}
M_i[n] &= \alpha_a M_{i-k_1,C}[n-1] + \alpha_d M_{i-k_2,C}[n-1] + \\
&\sum_{k=i-k_2+1}^{i-k_1-1} \alpha_{bc} M_{k,C}[n-1]\ ; \text{for } n = 2 : N_{NPT_{max}}
\end{aligned} \tag{11}$$

For ($n = 1$), the number of MTC devices, i.e. $M_i[1]$, will be the value determined by Beta distribution, detailed below.

C. Beta Distribution

Let M be the total number of MTC devices in the cell. By assuming that all the MTCs will be activated, according to Beta distribution, between ($t = 0$) and ($t = T$), the expected number of arrivals in the random access opportunity (i) is given by the following equation:

$$M_i[1] = M \int_{t_i}^{t_{i+1}} p(t)dt \tag{12}$$

where t_i is the time of the RA opportunity (i), and the distribution $p(t)$ follows Beta distribution:

$$p(t) = \frac{t^{\alpha-1}(T-t)^{\beta-1}}{T^{\alpha+\beta-1}Beta(\alpha,\beta)}\ ;\ \alpha > 0, \beta > 0 \tag{13}$$

where $Beta(\alpha, \beta)$ is Beta function, and it is given by:

$$Beta(\alpha,\beta) = \frac{(\alpha-1)!(\beta-1)!}{(\beta+\alpha-1)!} \tag{14}$$

It should be noted that $\int_0^T p(t)dt = 1$, and the values α and β are set to be (3) and (4), respectively, for MTC Beta traffic [8].

In order to find the expected number of arrivals at each RA slot, we approximate the integration in the equation (12) by using the trapezoidal rule [13]:

$$\int_a^b f(x)dx = (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

Therefore, the equation (12) can be written by:

$$M_i[1] = M(t_{i+1}-t_i)\frac{p(t_i)+p(t_{i+1})}{2} \tag{15}$$

As the interval between two consecutive RA slots is equal to T_{RA_REP}, we set $t_{i+1} - t_i = T_{RA\_REP}$, and therefore:

$$\begin{aligned}
M_i[1] = \frac{MT_{RA\_REP}}{2T^{\alpha+\beta-1}Beta(\alpha,\beta)} \Big[& t_i^{\alpha-1}(T-t_i)^{(\beta-1)} + \\
& t_{i+1}^{\alpha-1}(T-t_{i+1})^{(\beta-1)}\Big]
\end{aligned} \tag{16}$$

Equation (16) represents the number of new arrivals according to Beta distribution. Having determined the number of MTCs for each preamble transmission at the RA slot (i) (equations 11 and 16), we calculate the number of success MTCs by the equation (6), where the number of collided MTCs is $M_{C,n}(i) = M_i[n] - M_{S,n}(i)$. It should be noted that our analytical model GRE can be applied for another traffic models, where the only change is the number of new arrivals, i.e. $M_i[1]$, while the rest of the model remains unchanged.

IV. PERFORMANCE EVALUATION

The proposed model has been implemented using C++ language. The parameters of RACH procedure are taken as specified by Table 6.2.2.1.1 in [8]. Moreover, the control-plane latency analysis is determined as in Table B.1.1.1-1 in [14], where these parameters are specified in Table I. The simulations were developed based on Monte-Carlo approach, where 350 experiments have been used to average the results.

Notations	Definition	Values
$\boldsymbol{\alpha, \beta}$	The parameters of Beta Distribution	$3, 4$
$\boldsymbol{I_\beta}$	The interval of Beta Distribution	$10 * 1000$
$\boldsymbol{M}$	Average number of MTC devices in the cell	30000
$\boldsymbol{R}$	Total number of preambles in a random access slot	54
$\boldsymbol{BI}$	Backoff indicator in a sub-frame unit	20
$\boldsymbol{N_{PT_{max}}}$	Maximum number of preamble transmission	16
$\boldsymbol{N_{RAR}}$	Maximum number of RARs that can be carried in one response message	3
$\boldsymbol{T_{RAR}}$	Processing delay required by the eNB in order to detect the transmitted preamble in a sub-frame unit	2
$\boldsymbol{W_{RAR}}$	The size of the random access response window in a sub-frame unit	5
$\boldsymbol{N_{ACK}}$	Maximum number of MTC devices that can be acknowledged within the RAR window	$N_{ACK} = N_{RAR} \times W_{RAR}$
$\boldsymbol{PRACH_{config\_indx}}$	PRACH configuration index	$PRACH_{confic\_indx} = 6$
$\boldsymbol{T_{RA\_REP}}$	The interval between two consecutive Random Access (RA) slots	5
$\boldsymbol{W_{BO}}$	Backoff window size	$BI + 1$
$\boldsymbol{p_n}$	Preamble detection probability for the n-th preamble transmission	$p_n = 1 - e^{-n}$
$\boldsymbol{T_{CRT}}$	Contention Resolution timer	48
$\boldsymbol{p_{HARQ\_RET}}$	HARQ retransmission probability for Msg3 and Msg4 (non-adaptive HARQ)	10%
$\boldsymbol{N_{HARQ}}$	Maximum number of HARQ TX for Msg3 and Msg4 (non-adaptive HARQ)	5

TABLE I: Basic simulation parameters

A. *Performance Metrics*

In order to show the performance of our proposed model GRE, we will consider the following metrics: $i)$ the total number of MTC devices at each RA slot, $ii)$ the number of success MTCs at each RA slot, $iii)$ the collision probability, $iv)$ the success probability, $v)$ the average number of preamble transmission, $vi)$ and the Cumulative Distribution Function (CDF) of preamble transmission. The total number of MTCs is given by $M_i = \sum_{n=1}^{N_{PT\_max}} M_i[n]$, while the number of success MTCs is given by $M_S(i) = \sum_{n=1}^{N_{PT\_max}} M_{S,n}(i)$. Regarding the collision probability, it can be defined as the number of collided RAOs to the total number of reserved RAOs, and it is given by the following equation:

$$P_C = \frac{\sum_{i=1}^{I_{ra}} (R - M_S(i) - Re^{-\frac{M_i}{R}})}{RI_{ra}} \tag{17}$$

The success probability is equal to the number of success MTCs within the maximum number of preamble transmission to the total number of MTCs, and it is given by the following equation:

$$P_S = \frac{\sum_{i=1}^{I_{ra}} M_S(i)}{M} \tag{18}$$

Regarding the average number of preamble transmission, it is equal to the total number of preamble transmission for all the MTCs successfully accessed the network divided by the total number of success MTCs, and it is given by:

$$PRM_{avg} = \frac{\sum_{i=1}^{I_{ra}} \sum_{n=1}^{N_{PT\_max}} nM_{S,n}(i)}{\sum_{i=1}^{I_{ra}} \sum_{n=1}^{N_{PT\_max}} M_{S,n}(i)} \tag{19}$$

Let ω be the number of preamble transmission to access the network for the MTC devices successfully finished the RACH procedure. The CDF of preamble transmission, noted by $CPT(\omega)$, is the ratio between the number of MTC devices whose number of preamble transmission is less than or equal to (ω) and the total number of preamble transmission for all the MTC devices successfully accessed the network. CPT(ω) is given by the following equation:

$$CPT(\omega) = \frac{\sum_{i=1}^{I_{ra}} \sum_{n=1}^{\omega} M_{S,n}(i)}{\sum_{i=1}^{I_{ra}} \sum_{n=1}^{N_{PT\_max}} M_{S,n}(i)} \tag{20}$$

B. *Results*

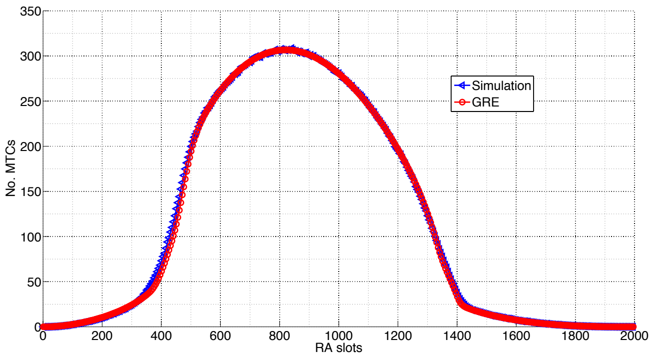

Fig. 3: The total number of MTC devices in each RA slot

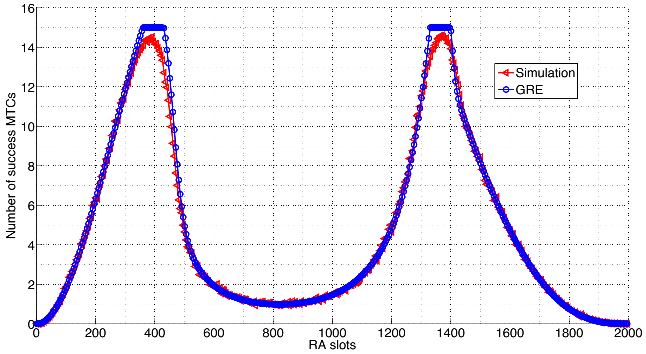

Fig. 4: The number of success MTC devices in each RA slot

Figure 3 shows the total number of MTC devices at each RA slot. From this figure, we see clearly that our analytical model GRE gives an accurate approximation of Beta Distribution. However, there is a small difference between the simulation and the analytical model when the total number of MTC devices is of the order of the number of preambles. This difference is clearer in figure (4), which represents the number of success MTC devices at each RA slot, where the accuracy of our analytical model is, generally, achieved except the regions where the total number of MTCs is of the order of the number of preambles. This difference comes from the fact that the network cannot send back responses to more than N_{ACK} MTC devices even if the number of success preambles is more than N_{ACK}. Therefore, when we take a lot of trails, the mean value will never reach the value N_{ACK} as it is the maximum allowed value, while in the analytical model we use the value N_{ACK} directly when the number of success preambles is more than N_{ACK}. This case is clear from figure (4), where the upper part of the analytical model is shown as if it is cut. Solving this problem will be one of the main our future works. In spite of this difference between the simulation and the analytical model, the results in Table II indicate that our analytical model has

	Success prob. (%)	**Collision prob. (%)**	**Avg No. preamble trans.**
Simulation	31.37	45.41	3.40
GRE	32.11	45.84	3.34

TABLE II: Comparison between the simulation and the analytical model

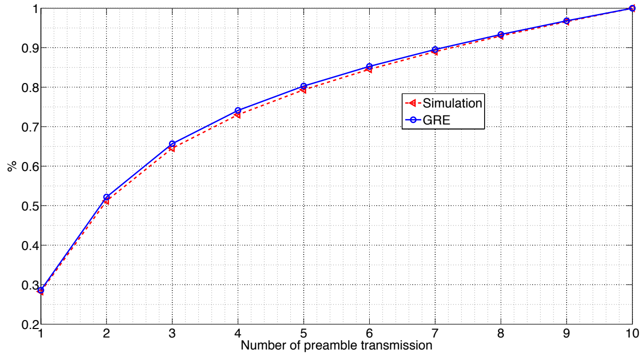

Fig. 5: CDF of preamble transmission

an accurate approximation regarding the success and collision probabilities, where the difference is less than one percent. Concerning the average number of preamble transmission, we also see that the analytical model gives a good approximation. The effectiveness of our analytical model is further proved by figure (5) that illustrates the Cumulative Distribution Function (CDF) of preamble transmission. It should be noted that our analytical model is valid for any traffic model, where only the number of new arrivals at each RA slot, i.e. $M_i[1]$, will be changed according to the traffic model while the rest of the model will be unchanged.

V. CONCLUSION

Beta Distribution is one of the traffic models proposed by 3GPP in order to evaluate the performance of the network under different access intensities. In this paper, an analytical model, namely General Recursive Estimation (GRE), has been proposed to model the RACH procedure in the existence of MTC traffic with Beta Distribution. In GRE many metrics have been considered, such as the total number of MTC devices at each RA slot and the CDF of preamble transmission. Simulation results show the accuracy of GRE. For example, the difference between the simulation and GRE, regarding the success and collision probabilities, is less than one percent. GRE also gives an accurate estimation of the average number of preamble transmission and the CDF of preamble transmission. The advantage of our analytical model GRE is that it could be used to model another types of traffic, where the only change to be done is the number of new arrivals while the rest of the model remains unchanged. Therefore, GRE is a general analytical model. Extending GRE to model another parameters, such as the power consumption, to include the Human-to-Human (H2H) traffic, and also to support the Access Class Barring (ACB) scheme will be one of our main future directions.

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